TWO-DIMENSIONAL PROBLEM OF THE DEFORMATION OF A PLATE BY A GLANCING DETONATION WAVE

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This paper is concerned with the nonsymmetric expansion of explosion products bounded on one side by a line of expansion into a vacuum and on the other by a plate whose shape is also not known in advance. When a metal plate is projected by a plane charge, the expansion of the explosion products is three-dimensional. However, when the length and width of the charge are much greater than its thickness, in the first approximation the effect of lateral expansion may be neglected and the problem becomes two-dimensional. The solution of the problem of the plane stationary supersonic motion of a gas has been obtained numerically on a computer by the method of characteristics with a preliminary calculation of the initial supersonic section by the power series method. The symmetrical problem of the expansion of explosion products has been examined by numerous authors, one of the earliest studies being that of Hill and Pack [1].

1. Statement of the Problem. In Fig. 1 region 1 is occupied by undetonated explosive, the detonation wave AB moves to the left at velocity D, region 2 is occupied by the expanding explosion products. We select a rectangular coordinate system moving together with the detonation wave. In this system the detonation wave is stationary, in region 1 gas of density ρ_0 moves to the right at velocity D, on the line AB the flow parameters are determined by the Chapman-Jouguet conditions, the flow velocity being equal to the speed of sound in the detonation products. The flow in region 2 is supersonic, the line AB being the sonic line.

Thus we have the following gasdynamic problem: to find the flow parameters in region 2 occupied by a polytropic gas. In this region the continuity equation, the irrotationality condition, and Bernoulli's equation, i.e., the equations of plane stationary irrotational gas motion in the absence of friction and heat conduction, are satisfied.

Assuming that the explosion products constitute a polytropic gas with adiabatic exponent k, we also know the pressure-density relation

$$p = \frac{D^2 k^k}{\rho_0^{k-1}(k+1)^{k+1}} \rho^k . \tag{1.1}$$

Furthermore, we know the relation between the flow parameters in region 1 and the parameters on the line AB

$$\rho_D = \frac{k+1}{k}\rho_0, \quad u_D = a_D = \frac{kD}{k+1}, \quad v_D = 0, \quad p_D = \frac{\rho_0 D^2}{k+1} .$$

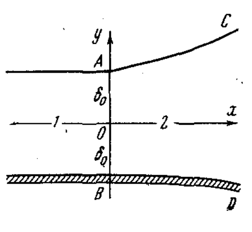

Fig. 1

The subscript 0 relates to the parameters in region 1, the subscript D to the parameters on the line AB. For the speed of sound from (1.1) we easily obtain

$$a^2 = a_D^2 (\rho/\rho_D)^{k-1} .$$

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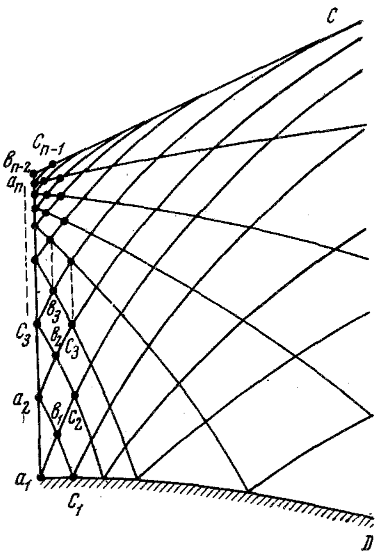

Fig. 2

We introduce the new unknown function $\varphi(x,y)$ or velocity potential, so that

$$u = \partial\varphi / \partial x, \qquad v = \partial\varphi / \partial y.$$

We now write the boundary conditions that must be satisfied by the flow parameters in region 2. We have:

on the line $x = 0$, $-\delta_0 \le y \le \delta_0$

$$p = p_D, \quad u = u_D = a_D, \quad v = 0, \quad \rho = \rho_D$$

at the free surface $y = g(x)$

$$p = 0, \qquad v - ug'(x) = 0$$

on the plate

$$v + u f'(x) = 0$$

(δ_0 is half the thickness of the slab of explosive).

Moreover, an additional boundary condition is imposed on the plate. This condition was obtained in [2] in the form:

$$\frac{f''(x)}{[1 + (f'(x))^2]^{3/2}} = \frac{p[x, f(x)]}{\rho_1 \delta_1 D^2}. \tag{1.2}$$

Here ρ_1 and δ_1 are respectively the density and thickness of the plate, $p[x, f(x)]$ is the pressure on the plate at the point $[x, f(x)]$. In this case the plate was treated as a layer of incompressible fluid. Moreover, it was assumed that the only force acting on an element of the plate is the pressure of the explosion products directed along the normal to the plate. The velocity of each element of the plate in a direction tangential to the curve $y = f(x)$ was assumed to be equal to the detonation velocity in the stationary coordinate system.

We go over to dimensionless parameters in accordance with the equations

$$\begin{gathered} x = \delta_0 x', \quad y = \delta_0 y', \quad u = a_D u', \quad v = a_D v' \\ a = a_D a', \quad \rho = \rho_D \rho', \quad p = p_D p', \quad \varphi = a_D \delta_0 \varphi'. \end{gathered} \tag{1.3}$$

In these equations the primed quantities denote dimensionless variables. We now obtain the following equations (the primes have been omitted):

$$\begin{gathered} \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \quad u^2 + v^2 = 1 - 2\int^{\rho} \frac{a^2 d\rho}{\rho} \\ a^2 = \rho^{k-1}, \quad u = \partial\varphi / \partial x, \quad v = \partial\varphi / \partial y \end{gathered} \tag{1.4}$$

with the boundary conditions:

on the line $x = 0$, $-1 \le y \le 1$

$$p = 1, \; u = 1, \; v = 0, \; \rho = 1 \tag{1.6}$$

at the free surface $y = g(x)$

$$p = 0, \qquad v - ug'(x) = 0 \tag{1.7}$$

on the plate $y = f(x)$

$$\begin{gathered} v + uf'(x) = 0, \\ \frac{f''}{(1 + f'^2)^{3/2}} = \frac{rp}{k+1} \quad \left(r = \frac{\rho_0 \delta_0}{\rho_1 \delta_1}\right). \end{gathered} \tag{1.8}$$

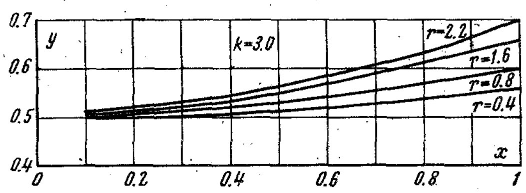

Fig. 3

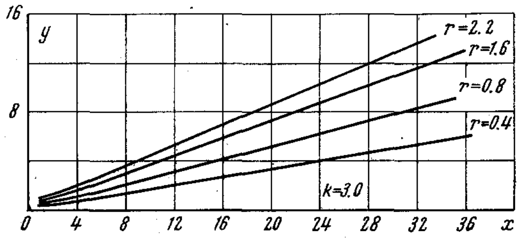

Fig. 4

A numerical solution of problem (1.4), (1.5) with boundary conditions (1.6)-(1.8) was obtained on a computer by the method of characteristics with a preliminary calculation of the initial supersonic section by the power series method.

2. Calculation of the Supersonic Gas Motion. The problem of calculating the initial supersonic section has been examined by many authors (see [1, 3, 5]). Our approach is based on the method proposed in [1]. Representing the unknown functions in the form of series in a neighborhood of the sonic line, from Eqs. (1.4) we obtain two infinite systems of equations for the coefficients of these series. Confining ourselves to small quantities of the third order, we obtain the solution in the form

$$u = 1 + 3P(y)x^2, \quad v = P'(y)x^3, \quad \rho = 1 - 3P(y)x^2 .$$

Here the function P(y) is determined from the equation

$$P'' = 18(k+1)P'^2$$

whose solution

$$P(y) = \sqrt[3]{-C_1}\,\wp\{\sqrt[6]{-C_1}\,[\sqrt{3(k+1)}\,y + C_2];\, 0, 1\}$$

is given by a Weierstrass function with real half-period $\omega_2 = 1.52995$; values of this function are tabulated in [4]. Starting from the boundary conditions at the free surface and on the plate and from the properties of the Weierstrass function, we uniquely determine the two constants C_1 and C_2.

Finally, we have

$$P(y) = \left[\frac{1.52995}{2\sqrt{3(k+1)}}\right]^2 \wp\left[\frac{1.52995}{2}(y+3);\, 0, 1\right].$$

The function P(y) has a pole at the point where the sonic line meets the free surface; therefore we still need to determine the flow in the neighborhood of that point. However, as shown in [1], this flow is described by the Prandtl-Meyer solution.

Thus, we first constructed a noncharacteristic line, conditions on which were then taken as the boundary conditions in calculating the motion of the gas by the method of characteristics.

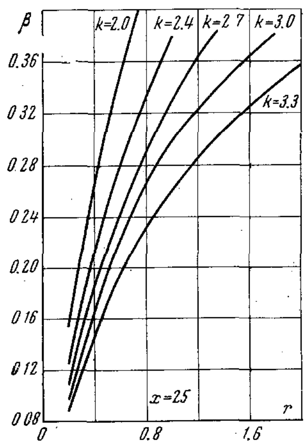

Fig. 5

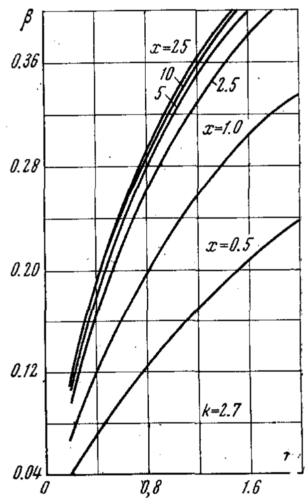

Fig. 6

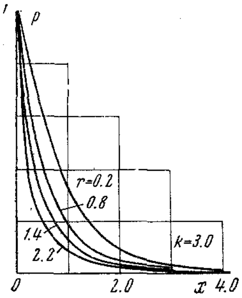

Fig. 7

In calculating the supersonic motion of the gas it is necessary to solve three elementary problems: a) calculation of a point in the flow field, b) calculation of a point on the plate, c) calculation of a point on the free surface. All these were solved by the standard methods described, for example, in [6].

3. Solution Algorithm and Results of the Calculations. The general network of characteristics is presented in Fig. 2. Here, $a_1, a_2, \ldots, a_n$ is the calculated noncharacteristic line, $a_n C$ is the free surface, and $a_1 D$ is the plate.

The flow parameters are known at the points $a_1, a_2, \ldots, a_{n-1}, a_n$. Solving problem (a) successively for the pairs of points a_1, a_2; a_2, a_3; ...; a_{n-2}, a_{n-1}, we find a certain auxiliary curve $b_1, b_2, \ldots, b_{n-2}$. Then, solving problem (b) for points b_1 and a_1, we find the point c_1 and establish the point c_{n-1} from the known point a_n on the free surface. We then determine the points $c_2, c_3, \ldots, c_{n-2}$, solving problem (a) for the pairs of points $b_1 b_2$; $b_2, b_3, \ldots, b_{n-3} b_{n-2}$, respectively. Thus, we obtain the flow parameters on a certain new noncharacteristic curve $c_1, c_2, \ldots, c_{n-1}$. After each new noncharacteristic curve is obtained, the process described is repeated. This algorithm enables the flow calculations to be carried to any length (depending on the number of given points on the noncharacteristic curve).

The program, written in Algol-60, contains three arbitrary parameters: k, the adiabatic exponent of the explosion products, r, the mass ratio of the explosive and the plate, and n, the number of known points on the noncharacteristic curve.

The results of the calculations are presented in graph form. Here, x and y are measured in units of charge thickness, and p in units of Chapman–Jouguet pressure. The shape of the plate in the plane of the variables x, y is shown in Figs. 3 and 4 for different values of r at $k = 3.0$. In Fig. 3 the angle of inclination of the plate β is plotted as a function of r at $x = 25.0$ for various values of the adiabatic exponent k. In Fig. 6 the β-r curves are plotted for various values of x at $k = 2.7$. Finally, Fig. 7 shows the dependence of the pressure p on the plate on x at $k = 3.0$ for various r. In all the figures the distance x is measured from the detonation front.

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